Hall Ticket Number:

Code No. : 16110 N(H)

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CBCS) VI-Semester Main Examinations, May-2019

Probability and Statistics for Engineers

(Open Elective-VII)

Time: **3** hours Max. Marks: **70**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Q. No.	Stem of the question	M	L	CO	PO
	Part-A (10 × 2 = 20 Marks)				
1.	State the properties of joint Cumulative Distribution function.	2	1,2	1	1
2.	Define conditional probability density function of X given Y = y.	2	1,2	1	1
3.	State the properties of Joint Characteristic Function.	2	1,2	2	1
4.	Find the relation between Co-variance and Variance.	2	1,2	2	1
5.	Define Population and Sample with examples.	2	1,2	3	1
6.	Define Estimator and Estimate.	2	1,2	3	1
7.	State the difference between Linear and Curve Linear Regression.	2	1,2	4	1
8.	What is Regression and Explain its importance.	2	1,2	4	1
9.	When did you say that two random variables are independent?	2	1,2	1	1
10.	What is likelihood function?	2	1,2	3	1
	Part – B (10 × 5 = 50 Marks)				
11.a)	Three balls are drawn at random without replacement from a box containing 2 white; 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn; find the joint probability distribution of (X, Y).	5	1,2	1	1
b)	Two-dimensional random variable (X, Y) have the joint density $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & otherwise \end{cases}$ i) Find P (X < ½ ; Y < ¼). ii) Find the marginal and conditional distributions. iii) Are X and Y independent? Give reasons for your answer.	5	3	1	1
12. a)	Two random variables X and Y have the following joint probability density function: $f(x, y) = \begin{cases} 2 - x - y; & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0, & otherwise \end{cases}$ Find i) Marginal probability density functions of X and Y; ii) Conditional density functions; iii) Var (X) and Var (Y) ; and iv) Covariance between X and Y.	5	3	2	1

Contd... 2

b) The joint probability distribution of X and Y is given by the following table: | 5 | 1,2 | 2 | 1

Y \ X	1	3	9
1	1/8	1/24	1/12
2	1/4	1/4	0
53	1/8	1/24	1/12

i) Find the marginal probability distribution of Y.
ii) Find the conditional distribution of Y given that X = 3.
iii) Find the covariance of X and Y.
iv) Are X and Y independent?

13. a) Estimate α and β by the method of moments: | 5 | 4 | 3 | 1

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x},\ 0\le x<\infty$$

b) The sample values from population with p.d.f: | 5 | 4 | 3 | 1

$f(x)=(1+\theta)x^{\theta},0<x<1,\theta>0$, are given below :

0.46, 0.38, 0.61, 0.82, 0.59, 0.53, 0.72, 0.44, 0.59, 0.60. Find the estimate θ by
i) method of moments and
ii) maximum likelihood estimation.

14. a) The expected remaining life of an electronic part is believed to be related to the age of the part. The ages of 10 of these parts that were in use on a certain date were recorded in operating hours. When each part burned out, the elapsed time was recorded. The results were as follows: | 5 | 1,2 | 4 | 1

Age of Part (in hrs) : 40 65 90 5 30 10 80 85 70 25
Remaining life(in hrs): 30 20 10 80 40 65 15 15 20 50

Determine the regression curve using the non-linear form $Y = ab^X$

b) Define curve of regression of Y on X. the joint density function of X and Y is given by: | 5 | 3 | 4 | 1

$$f(x,y)=\begin{cases}x+y, 0<x<1, 0<y<1\\ 0, \textit{otherwise}\end{cases}$$

Find
i) the regression curve of Y on X, and
ii) the regression curve of X and Y.

15.a) For bivariate probability distribution of (X; Y) given below; find | 5 | 1,2 | 1 | 1
$P(X \le 1)$; $P(Y \le 3)$; $P(X \le 1; Y \le 3)$; $P(X \le 1/Y \le 3)$; $P(Y \le 3/X \le 1)$ and $P(X+Y \le 4)$.

Y \ X	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Contd... 3

b)	A fair coin is tossed four times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. i) Determine the joint distribution of X and Y; and ii) Find Cov (X, Y).	5	3	2	1
16.a)	X_1, X_2, and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2. T_1, T_2, and T_3 are the estimators used to estimate mean value μ, where $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_2 - 4X_3$, and $T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)/3$. i) Are T_1 and T_2 unbiased estimators? ii) Find the value of λ such that T_3 is unbiased estimator of μ. iii) With this value λ is T_3 a consistent estimator? iv) Which is the best estimator?	5	1,2	3	1
b)	The job rating efficiency of an employee seem to be related to the number of weeks of employment. For a random sample of 10 employees, the following data were observed: Job efficiency (X) 55 50 20 55 75 80 90 30 75 70 Weeks of employ-ement(Y): 2 4 1 3 5 9 12 2 7 5 Determine the coefficients of regression and regression equation using the non-linear form $Y = a + b_1 X + b_2 X^2$	5	1,2	4	1
17.	Answer any ***two*** of the following:				
a)	Two trains arrive at a station at random between 7 A.m. and 7.30 A.m. one train stops for 5 minutes and the other for x min. for what value of x; will the probability that the 2 trains meet be equal to 1/3?	5	4	1	1
b)	The amount of money spent on research and development (R and D) by a large corporation is believed to have an effect on their gross sales. For the past 12 years, the following data have been recorded: Determine the coefficients of regression and regression equation using the non-linear form $Y = a + b_1 X + b_2 X^2$	5	1,2	4	1

Year Number	Amount Spent on R and D (in Rs. 100,000's)	Gross Sales (in Rs. 10,00,000's)
1	1.9	2.8
2	3.4	3.2
3	6.5	3.0
4	5.7	4.0
5	2.8	4.6
6	2.3	3.8
7	6.2	3.4
8	7.6	3.2
9	5.0	4.7
10	5.3	5.2
11	4.7	2.8
12	5.2	5.0

Contd... 4

c) The joint probability distribution of a pair of random variables is given by the following table:	5	1,2	2	1

X / Y	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Find:

i) The marginal distributions.

ii) The conditional distribution of X given Y = 1.

iii) P { (X + Y) < 4 }.

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

S. No.	Criteria for questions	Percentage
1	Fundamental knowledge (Level-1 & 2)	70
2	Knowledge on application and analysis (Level-3 & 4)	30
3	*Critical thinking and ability to design (Level-5 & 6) (*wherever applicable)	-